CHAPTER 4

MASS BALANCES, LOADING RATES, AND FISH GROWTH

4.0 INTRODUCTION

Water flow is the mechanism by which oxygen is transported into a fish culture vessel and the waste products being generated within are removed. The design of a recirculating aquaculture system (RAS) should insure that the important parameters affecting water quality and fish productivity, e.g., oxygen, ammonia, carbon dioxide, and suspended solids are properly balanced. This requires calculating the value of each of these parameters independently to determine the thresholds for each. Then, having done the necessary calculations, the system must be operated at the highest flow rate possible while still maintaining a particular parameter at or below it's maximum tolerable or design value, e.g., ammonia. Obviously, the maximum flow rate possible while maintaining one particular parameter may be too high for maintaining another. The same mass balance approach can be utilized on any variable affecting water quality. It simply comes down to balancing the transport in, the production of a particular parameter within the culture tank, and the transport out. In word equation form, we like to say:

*Transport **in** of "x" + **production** of "x" =*
*transport **out** of "x"* (4.1)

The production term can be the production of oxygen, ammonia, suspended solids, or CO_2. Note that the production term can be negative, meaning consumption of a certain component, e.g., oxygen.

We can depict a mass balance for the general case where part of the flow is recirculated and part of the flow is flow-through as:

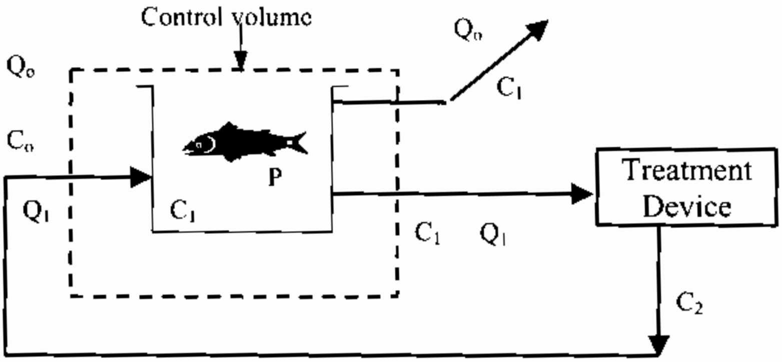

Figure 4.1 General Mass Balance on a Fish Culture Tank. Treatment occurs exterior to the tank.

We are assuming a completely well mixed tank and that the tank has reached a non-changing condition with respect to time or steady-state conditions. The box outside the fish tank represents some treatment device or process that changes the concentration of the noted parameter "x". (Note: there could be several treatment devices, each treating a different water quality variable.)

Returning to our word equation, Eq. 4.1 and Fig. 4.1, we can perform the mass balance assuming steady state conditions:

$$Q_1 C_2 + Q_0 C_0 + P = Q_0 C_1 + C_1 Q_1 \qquad (4.2^{a})$$

To obtain accurate and reliable results from these equations, it is essential that each of the terms or products of terms in Eq. 4.2 is represented by the same unit value, e.g.,

$$\frac{kg_{oxygen}}{day}$$

For example, the unit balance example for a transport of oxygen flowing into the tank would be:

$$\mathrm{QC} = \frac{kg_{water}}{day} \quad \frac{kg_{oxygen}}{kg_{water}} \quad = \quad \frac{kg_{oxygen}}{day} \qquad (4.3)$$

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[a] Symbols are defined at end of chapter.

If we were doing a mass balance using Eq. 4.2 for oxygen, then all terms or products would need to have the same units of kg oxygen per unit time. It is convenient to use "day" as the time unit, since growth rates and feeding rates are generally measured on a per-day basis. BE CAREFUL to be consistent with unit designations.

Transport is the key term in these calculations, and it is defined as the product of flow and concentration. For example, the remainder of oxygen transport into the tank minus the allowable minimum level of oxygen departing the tank defines the oxygen available for fish growth. Flow is measured as volume per time or mass per time, and will be usually defined in terms of:

- gallons per minute, gpm
- liters per minute, Lpm
- kg per sec, kg/s
- m^3/s

Typically, most water quality parameters are expressed in terms of:

$$\frac{mg}{Liter} \quad or \quad \frac{mg}{L}$$

The usage of mg/L is often called or referred to as:

ppm or parts per million.

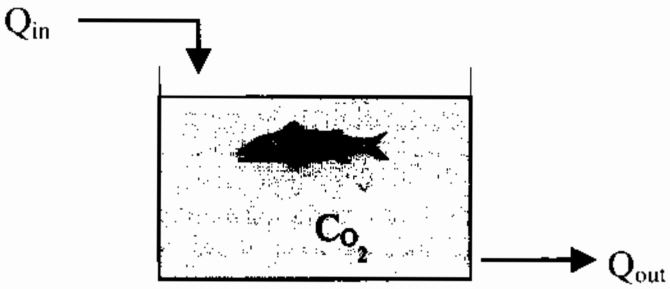
These values are the same.

This usage comes from:

$$\frac{mg}{L} \cdot \frac{L}{1000g} \cdot \frac{g}{1000mg} = \frac{1}{10^6} \quad or \quad \frac{1\ part}{million} \text{ or } 1\ ppm$$

Thus: $$10\frac{mg}{L}\ \ oxygen \quad \rightleftarrows \quad 10\,ppm\ \ oxygen$$

Now, to reinforce your understanding, let's calculate the available oxygen to support fish growth assuming a mass flow of water of 100 gpm of saturated inlet water at 60°F at an elevation 800 feet above sea level.



$Q_{in}\,C_{in}$ = O_2 transported in
$Q_{out}\,C_{tank}$ = O_2 transported out

Equation 4.4 provides an approximation that can be used to estimate oxygen concentration in non-saline waters (Chapter 8 provides a more detailed approach to determine gas solubility including oxygen, carbon dioxide, nitrogen, and argon):

$$C_{saturation}\,\frac{(mg)}{L} = \frac{132}{(T\,^{\circ}F)^{0.625}} \cdot \frac{760}{760 + \dfrac{altitude(ft)}{32.8}} \tag{4.4}$$

Using Eq. 4.4:

Altitude of 800 ft.
Temperature 60°F

$$C_{sat} = 10.21 \cdot 0.97 = 9.89\,\frac{mg}{L}$$

For the mentioned flow rate of 100 gpm of water and with the incoming water having a concentration of 9.89 mg/L, the mass of oxygen transported into the tank on a daily basis is ("gpm" units are used since they are common terminology in US):

$$Q_{in}\,C_{in,\,oxygen} = 100\,\frac{gal}{min} \cdot 9.89\,\frac{mg}{L}$$

Now, make the units consistent:

$$= 100\,\frac{gal}{min}\cdot 3.785\,\frac{L}{gal}\cdot 9.89\,\frac{mg}{L}\cdot 1440\,\frac{min}{day} = \frac{mg}{day}O_2$$

$$= 5{,}390{,}445.\frac{mg}{day}\cdot\frac{kg}{10^6\,mg} = 5.39\,\frac{kg\ O_2}{day}$$

This then is the oxygen available to the culture vessel on a daily basis to support fish growth and bacteria action. However, the water leaving the tank must still be at the minimum level necessary to support fish growth, e.g., 5 mg/L, so only a portion of the total available oxygen can be used.

The available oxygen is then (same as previous example except now a C_{out} for the discharge water is defined):

$Q\ (C_{in} - C_{tank}) =$

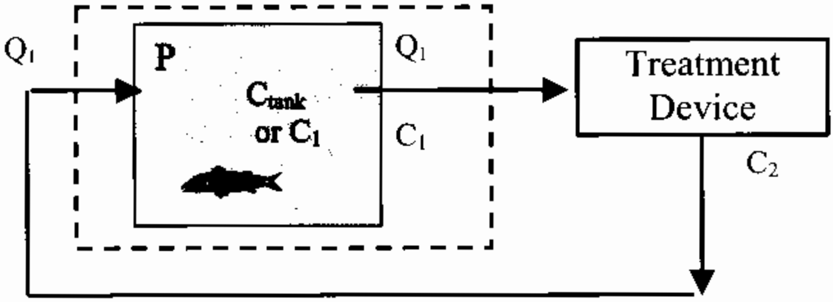
$$= 100\,\frac{gal}{min}\cdot 3.785\,\frac{L}{gal}\cdot (9.89 - 5)\frac{mg}{L}\cdot\frac{kg}{10^6\,mg}\cdot 1440\,\frac{min}{day}$$

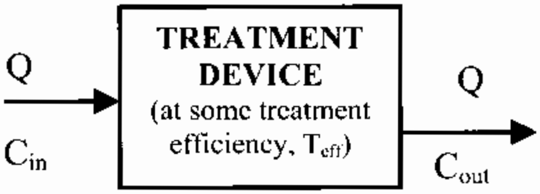
$$= 2.66\,\frac{\text{kg}}{\text{day}}\,\text{O}_2$$

Returning to the general mass balance Eq. 4.2, the simplest RAS case is where all water flow is recirculated and there is no discharge. In this situation, the Q_0 terms in Eq. 4.2 drop out $Q_{in} = Q_{out}$ have:

$$Q_1\,C_2 + P = +\,Q_1\,C_1 \tag{4.5}$$
$$Q_1\,(C_2 - C_1) = -\,P$$

Solving the general mass balance equation with the "magical" treatment box, we need to determine the concentration of each particular water quality parameter leaving the treatment device, so that the mass flow of the water quality parameter into the fish culture tank can be determined and the mass balance solved (C_2). Looking now at only the treatment device depicted in Fig. 4.1, the treatment box could be a biofilter, a CO_2 stripper, or a solids settling chamber. Each will have its own treatment efficiency for the particular water quality parameter it is designed to treat. We just use the box as a symbolic depiction of this treatment device.





Looking at this treatment box and doing a mass balance on the control volume, you can solve for the leaving water quality concentration C_{out}. Since the water flow in equals the water flow out, the C_{out} term can be solved for directly:

$$C_{out} = C_{in} + \frac{T}{100}\left(C_{best} - C_{in}\right) \tag{4.6}$$

where, C_{best} is the absolute best result obtainable by a treatment system, e.g., zero ammonia or saturated oxygen, zero suspended solids.

We have adapted the term "C_{best}" to represent what the treatment device is trying to do. If you had the perfect treatment device, the device is still limited by basic physical laws as to what it can achieve. Thus, the reference to *best*. Note that if the device is an oxygen addition unit, the C_{best} term can be increased above atmospheric concentration values for oxygen by increasing the partial pressure above atmospheric oxygen partial pressure in the device. For example, a pure oxygen device will have a C_{best} value of roughly five times the C_{best} value that is available if normal air used at atmospheric pressure, e.g., trickling tower. C_{best} for most other parameters should be fairly obvious to the reader, e.g., ammonia and TSS are zero, but CO_2 will be around 0.5 mg/L since there is some CO_2 in the air.

LOADING RATES

The relationships of production capacity to flow exchanges and space are important to aquacultural engineers. The terminology "loading" (L) is used to describe the fish mass that can be maintained per unit of flowing water, kg fish per liter per minute flow (kg/Lpm). Fish density (D_{fish}), defined as kg fish per cubic meter of space (kg/m^3), combined with the number of water exchanges per hour (R) through the rearing unit produces the loading rate (L):

$$L = 0.06\,\frac{D_{fish}}{R} \tag{4.7}$$

The constant 0.06 in Eq. 4.7 converts Lpm to m^3/hr (1.0 Lpm · 60 min/h = 60 L/h or 0.06 m^3/h, there are 1,000 L in a m^3). The loading capacity depends primarily on water quality, fish size and species. Using Eq. 4.7 and assuming that fish metabolism requires 250 grams of oxygen for each kg of feed fed (no allowance for nitrification), the allowable loading (kg of fish per Lpm of flow) due to oxygen constraints is:

$$L_{oxygen} = \frac{144\ \Delta O_2}{250 F_{\%}} \tag{4.8}$$

The ΔO_2 term in Eq. 4.8 represents the allowable drop in oxygen from inlet to outlet. For example, if the inlet DO is 11.0 mg/L and the outlet is designed to maintain oxygen above 5 mg/L, then the ΔO_2 is 6.0 mg/L. Eq. 4.8 is demonstrated in Fig. 4.2 for various feeding rates and allowable changes in oxygen concentration.

As fish densities increase, minimum oxygen levels are better maintained by using pure oxygen systems, which permit inlet oxygen concentrations to be maintained at multiples of dissolved atmospheric concentration levels (see Chapter 8). However, other water quality parameters will begin to limit the allowable carrying capacity due to degradation in accumulated ammonia or carbon dioxide or suspended solids. The term for this condition is the cumulative oxygen consumption (COC) level. From basic stoichiometry and the production values for ammonia, solids, and carbon dioxide associated with feeding, then:

- 10 mg/L of oxygen consumed will produce up to 1.4 mg/L of ammonia,
- 14 mg/L of carbon dioxide, and
- 10 to 20 mg/L of suspended solids.

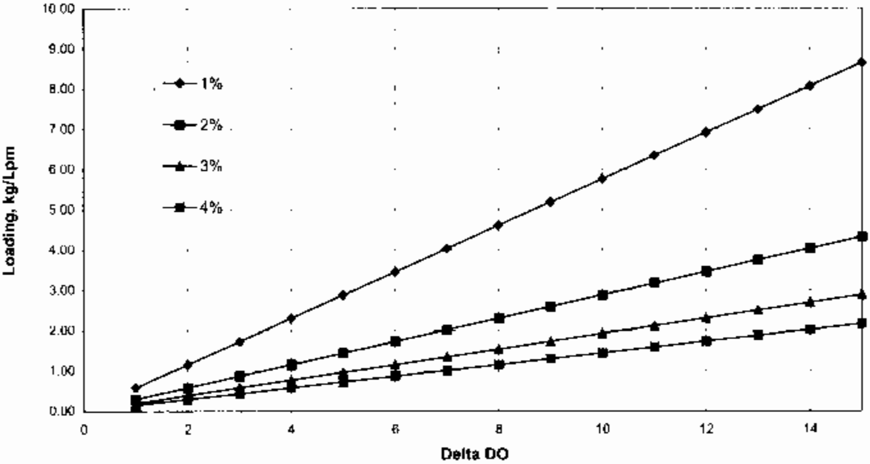


Figure 4.2 Allowable Fish Loading in kg per Lpm. Based upon the change in dissolved oxygen from inlet to outlet (Delta DO) and the percent body weight per day for the fish feeding rate (to convert kg/Lpm to lb per gpm, multiply by 8.33).

This relationship must be kept in mind when designing the fish culture vessel.

Maximum allowable or safe densities (D_{fish}) are much more difficult to ascertain than are maximum loadings. It still seems to be a very subjective process and has much controversy. Density is primarily a function of fish size, species, and the characteristics of the rearing environment and management skill. New growers tend to overestimate their own safe loading densities and assume they can establish and sustain densities from the very beginning that in fact require expert management skills. Do NOT fall into this trap. You will kill fish. New growers should target about 1/2 the densities recommended in this book for expert growers. This subject is discussed in detail in Chapter 6, Culture Tank Design.

4.1. PRODUCTION TERMS

The "P" term in Eq. 4.2 represents the production of some pollutant or consumption term. These terms in an RAS system can all be proportionately related to the fish feeding rate. In principle, if you are not feeding the system, there is no pollution. This generalization is valid, because even ammonia production rates reduce 10 fold within a day or so, once feeding activity has ceased. Oxygen consumption is also reduced by approximately 50% and fecal production goes to zero. However, the idea in any production system is to grow the animals, and to grow, they must be fed, so we relate the production terms exclusively to feeding rates as follows:

P_{Oxygen} = - 0.25 kg per kg feed consumed by fish (a negative production term)
- 0.12 kg per kg feed consumed by nitrifying bacteria
- 0.13 heterotrophic bacteria (estimate, can be as high as 0.5)
= - 0.50, kg (0.25 + ·12 + ·13) per kg feed for system including nitrifying and heterotrophic bacteria

P_{CO2} = 1.375 grams produced for each gram O_2 consumed (both fish and bacteria)

P_{TAN} = F · PC · .092

$P_{Solids,\ TSS}$ = 0.25 · kg feed fed (dry matter basis)
(literature gives values from 20% to 40% of feed fed on dry basis)

4.2 WATER QUALITY DESIGN TARGETS

A difficult concept to grasp for most is that in performing the mass balances, the designer/manager must choose design or target operating conditions. These are the "C" values in Eq. 4.2 shown in the culture tank. These design numbers are species dependent and are continually being refined for RAS applications. Our recommendations are given in Table 4.1 for a common warm (tilapia) and cool (trout) water species.

Table 4.1 Water Quality Parameters for a Cool and Warm Water Species

Parameter	Tilapia	Trout
Temperature, °F (°C)	75 to 85 (24 to 30)	50 to 65 (10 to 18)
Oxygen, mg/L	4 to 6	6 to 8
Oxygen partial pressure, mm Hg	90	90
CO_2, mg/L	40 to 50	20 to 30
Total suspended studies, mg/L	<15	<10
Total ammonia - N, mg/L	<3	<1
NH_3-N	<0.6	<0.02
Nitrite-N	<1	<0.1
Chloride, mg/L	>200	>200

SELECTING TARGET VALUES FOR WATER QUALITY

Calculating the minimum flows required to maintain targeted values for water quality (and then using the largest minimum value found for all the different water quality variables) will show how sensitive the calculated flow rates are to the value selected for the design value. A typical scenario is to select a value, do the calculations, realize that there is no way you could afford to supply such a high flow rate, and then start to make adjustments in the targeted values, e.g., 4 mg/L oxygen is probably OK instead of the 6 mg/L you originally chose, etc, etc. In the end, one must choose realistic values and then stay with these choices and the ramifications of the resulting flows required to maintain the mass balances. Do NOT ever compromise on the required flow rates. You will be sorry if you do.

OXYGEN

The major reason most fish die is from lack of oxygen due to a loss of water flow. This is because oxygen is consumed at a fairly high rate (fish metabolism) and oxygen is transported by water flow. Due to low inherent concentrations of oxygen, "high" flows are required to transport the required oxygen. Flows required to maintain a satisfactory oxygen level are generally the controlling flow rate parameter when solving the series of mass balance equations to determine the most restrictive parameter. Even a partial loss of flow will generally result in insufficient oxygen for the fish, resulting in death. And since this is a text on RAS, remember this rule about why fish die:

"RULE OF THUMB"

Loss of Water = Loss of Fish
Loss of Flow = Loss of Fish

Effective methods of monitoring and control are discussed in Chapter 9 in detail, but remember the above rule; always sense and monitor these two activities in at least two independent ways. If you don't, we guarantee that you will lose fish and eventually you will lose fish catastrophically.

Oxygen concentrations versus temperature and salinity are provided in the Appendix. For salmonids, as a group, the rearing unit effluent should contain from 6.0 to 8.0 mg/L dissolved oxygen (DO). For catfish and tilapia, allowable minimum levels are much lower than for salmonids, e.g., 2 or 3 mg/L, while it is certainly recommended to stay much closer to 5 or 6 mg/L. This variation in what the allowable minimums are has to do with the fact that partial oxygen pressure ($p0_2$) appears to be a more valid way to determine the lower limits. A $p0_2$ of 90 mm Hg seems to be a reasonable target for salmonids (Downey and Klontz, 1981). The atmosphere contains 21% oxygen, and at standard pressure of 760 mm Hg, this represents a $p0_2$ of 0.21 • 760 or 160 mm Hg or 56% of saturation. Where dissolved oxygen saturation represents 9.0 mg/L, 90 mm Hg represents 5.1 mg/L DO, and where saturation represents 12.5 mg/L DO, 7.0 mg/L represents a pO_2 of 90 mm Hg. These ranges represent temperatures from 20°C down to 5°C at sea level elevation (760 mm Hg). As noticed, the warmer the water the lower the effluent DO in mg/L can go, while still representing a pO_2 of 90 mm Hg. Applying 90mm pO_2 @ 30°C for tilapia would set a target value of 4.2 mg/L for culture tank oxygen levels (consistent with Table 4.1).

For salmonids and using 90 mm Hg as the minimum target, the available oxygen at 5°C is 5.5 mg/L (12.5–7.0 mg/L), at 20°C it is 3.9 mg/L (9.0–5.1 mg/L). Thus, considerably less oxygen is available at higher temperatures when metabolic rates, too, are higher, significantly impacting production capacity on the basis of limited available oxygen. Note the dilemma between higher temperatures supporting higher fish growth and metabolism, but the increasing unavailability of oxygen in the water column to support growth.

Beyond the general rule that each unit of feed will require 0.25 units of oxygen for fish metabolism, usage rates will depend mostly upon the type of fish being considered. Westers (1979) uses 200 to 250 g per kg of feed fed, while Pecor (1978) recommends using 110 g per kg of feed fed for esocids, such as the tiger muskellunge, a coolwater, non-active fish. Huisman (1975) uses 230 g for the common carp, as a warmwater representative. These values are consistent with our general recommendation of 250g O_2 per kg of feed fed for fish oxygen needs.

Finally, remember that not only do the fish require oxygen, but also the biological filter is just as critically dependent upon adequate oxygen levels to support bacteria metabolism. The DO concentration within the filter must be maintained at or above 2.0 mg/L to insure that the rate of nitrification in the filter does not become limited because of oxygen depletion (Kumar, 1984; Manthe et al. 1988). Always measure the DO coming off the biofilters and if the concentration starts to approach 2.0, then take corrective action, e.g., increase flow rate through the biofilter by increasing the hydraulic loading rate on the filter.

AMMONIA

There is considerable confusion about ammonia. Definitive values for the toxic levels of ammonia and the differentiation between the toxic NH_3 form and the supposed non-toxic NH_4^+ have never been determined. Meade (1985) reviewed the published literature on the effects of ammonia on fish and concluded:

> A truly safe, maximum acceptable concentration of un-ionized, or of total ammonia, for fish culture systems is not known.

The apparent toxicity of ammonia is extremely variable and depends on more than the mean or maximum concentration of ammonia.

The European Inland Fishery Advisory Commission (EIFAC) of FAO has set 0.025 mg/L as the maximum allowable un-ionized ammonia (NH_3 or A_{NH3-N}). Note, this means that tank ammonia levels (TAN) can exceed 10 mg/L if pH is maintained below 7.0. We would be very uncomfortable using such a high design value. What if the pH rose to 7.3 and doubled your NH_3? As seen in Table 4.1, we use rule of thumb values of 1 mg/L TAN for cool water and 2 or 3 for warm water fish. You should always check your TAN target value selection by assuming some pH that you intend to maintain and see if your NH_3 concentrations will exceed the 0.025 mg/L value using the ammonia-ammonium pH Temperature tables from the Appendix. If so, rethink your design carefully before proceeding.

There seems to always be a disproportionate amount of attention applied to nitrification and ammonification. All these calculations depend upon the estimate of an ammonia generation load, which is based upon the fish feeding rate:

$$P_{TAN} = \frac{F \cdot PC \cdot 0.092}{t = 1\ day} \qquad (4.9)$$

The constant in the ammonia equation is based upon a series of approximations and estimates that when multiplied together result in the 0.092.

$0.092 = .16 \cdot .80 \times .80 \cdot .90$

- 16% (protein is 16% nitrogen)
- 80% nitrogen is assimilated
- 80% assimilated nitrogen is excreted
- 90% of nitrogen excreted as TAN + 10% an urea (fresh water fish only)
- all TAN is excreted during time period "t"
- non assimilated nitrogen in feces is removed quickly
- (no additional mineralization of nitrogenous compounds)

The individual assumptions about digestion and ultimate production of ammonia that is diffused across the gill and excreted directly via the feces all lack crispness in their assignment. Thus, we tend to see the rate of ammonia generation as being a "soft" number. For simplicity, one could simply assume 10% of the protein in the feed becomes the ammonia-N generation rate.

Equation 4.9 represents a conservatively high estimate of the P_{TAN} production rate. We use the time period in Eq. 4.9 as one day, while others will use the time period between feedings. In RAS, feed can be fed uniformly over a 24 hour period, thus distributing the ammonia load uniformly over the entire day as well. If a uniform 24 hour feeding is not used, then the equation should be adjusted and the time period should be the time between feedings or if a single feeding per day is used, then use 4 hours as the time period as an estimate of the time for the ammonia to be excreted from a feeding event. The assumption that all of the TAN is excreted in a finite period of time (t) between feedings is founded in evidence that metabolic activity increases during the hours following feedings (Page and Andrews, 1974; Ruane et al. 1977). Although the value of t is dependent on many biological variables, experience has indicated that fish metabolic activity peaks from 1 to 4 hours following feeding. One quickly will conclude that many smaller feedings evenly spaced during the day would serve to minimize high values of P_{TAN}. In fact, this is a strategy employed in the production of fish in tanks through the use of automatic feeders or demand type feeders.

CARBON DIOXIDE, CO_2

Carbon dioxide is an important, but largely overlooked water quality limiting parameter. This is probably because until recently, most systems were generally low density (less than 40 kg/m^3) and relied on aeration as the main means of supplying oxygen. This type of management also kept CO_2 values at low levels, e.g., less than 20 mg/L. However, loading rates have increased in recent years, and it became necessary to inject pure oxygen into these systems, instead of using aeration. As result, the natural stripping of CO_2 that occurs when using aeration systems was no longer taking place. We now need to apply other means for CO_2 control.

The generation of CO_2 is based upon chemical stoichiometry by relating CO_2 production to oxygen consumption (1.375 is the ratio of molecular weights of the two gases, 44/32):

$$CO_2 = 1.375 \text{ grams produced for each gram } O_2 \text{ consumed (both fish and bacteria)}$$

Please refer to Chapter 8 for more extensive discussion on carbon dioxide control. Additionally, please note that a computer software program to design CO_2 removal systems is provided in Chapter 17. The relationship between CO_2 and alkalinity is shown in Fig. 4.3 for an alkalinity value of 100 mg/L. CO_2 concentration is proportional to alkalinity within normal ranges of pH encountered in aquaculture applications, e.g., pH of 6 to 8.5.

The available data on acceptable levels of CO_2 are difficult to interpret, at best. The authors have maintained tilapia systems in excess of 100 mg/L on a sustained basis, but this seems to be the exception. Colt and Watten (1988) recommend that the maximum level of CO_2 should not exceed 20 mg/L, while Needham (1988) recommends the maximum not to exceed 10.0 mg/L. Alabaster et al. (1957) report that in well aerated water, CO_2 levels don't become toxic for rainbow trout under 100 mg/L. On the other hand, 10 mg/L caused mortalities at a pH level of 4.5, and mortalities occurred at 20 mg/L CO_2 when the pH was 5.7 (Lloyd and Jordan, 1964). In other research, Piper et al. (1982) state that 40 mg/L CO_2 had little affect on juvenile coho salmon. However, they also mention that CO_2 in excess of 20 mg/L may be harmful to fish, and where DO levels drop to 3–5 mg/L, lower concentrations may be detrimental and long term exposure of one year or more should not exceed 12 mg/L. Smart (1981) argues that fish are able to acclimate to elevated levels of CO_2. High CO_2 levels may lead to nephrocalcinosis, the presence of white calcareous deposits in the kidney. The severity of this condition appears to vary greatly according to diet and environmental factors as well and in particular the form of alkalinity replacement used. Chen et al. (2001) reported that the usage of agricultural grade limestone in place of sodium bicarbonate in an intensive tilapia system led to increased levels of nephrocalcinosis.

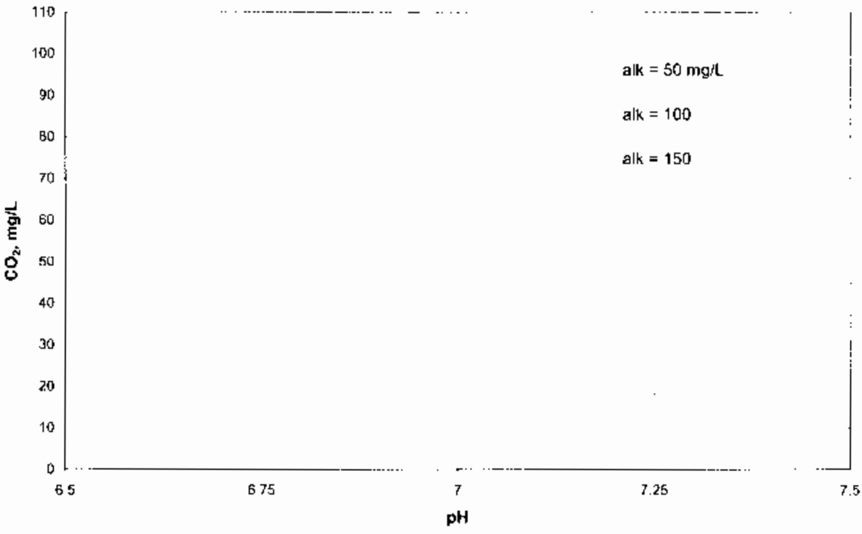


Figure 4.3 Concentration of CO_2 Versus pH at Alkalinities of 50, 100 and 150 mg/L.

A key point in the above discussion is that fish successfully acclimate to slowly changing water quality conditions, such as increases in the level of CO_2, but are adversely affected by sudden changes of water condition. Using CO_2 as an example, fish acclimated to CO_2 levels of 20 mg/L may die if exposed to a sudden spike to 80 mg/L CO_2 while other fish may exhibit good growth and feed conversion ratios at a gradually created but sustained level of 80 mg/L CO_2. An advantage of RAS is that system water quality can be maintained at fairly uniform values in a normally functioning system, but failures or malfunctions of the system that cause certain water quality parameters to fluctuate widely may cause fish to suffer or die.

SUSPENDED SOLIDS

The effective control of solids generated in RAS is probably the most important task that must be accomplished to ensure long-term successful operation of an RAS. This aspect of RAS is discussed in more detail in Chapter 6, but is also presented here as the suspended solids are a component of water quality. The quantity of suspended solids or Total Suspended Solids, TSS, generated per unit of feed being fed is estimated as:

$$\text{TSS} = 0.25 \cdot \text{kg feed fed (dry matter basis)}$$
$$\text{(from 20\% to 40\% of feed fed dry basis)}$$

TSS is treated as a dilute waste. TSS design concentrations in RAS will be in the 10 to 30 mg/Liter range. Even after concentrating TSS with some type of treatment process, a certain volume of water will still contain only around 0.5 to 1% solids on a dry matter basis. In comparison, cow manure is 20% solids. TSS captured in a settling basin has a fluffy consistency and will require substantial volumetric space depending upon frequency of cleaning. As a "rule of thumb", assume that each kg of dry feed fed will produce approximately 8 liters of liquid waste, i.e., one lb feed produces one gallon of manure.

NITRATE

Nitrate nitrogen (NO_3) is the end product of the nitrification process. In general, concentrations of nitrate are not extremely adverse to RAS water quality. We have maintained some salmonid systems at nitrate levels above 1,500 mg/L without impact on the fish. Nitrogen should be conserved throughout the nitrification process. Thus, if 1 kg per day of TAN is being produced, then 1 kg of nitrate-N is being produced. The equilibrium concentration of nitrate will therefore be directly dependent upon the overall water exchange rate through the system. An effective exercise is to calculate the steady state nitrate-N balance assuming some water exchange rate through the system (see Eq. 4.4). Nitrate-nitrogen is relatively non-toxic to fish and as such will not influence the controlling flow rates in the system. One can choose some value such as 500 mg/L if you want a number to work with.

4.3 FISH GROWTH

The premise of RAS design is that we are endeavoring to grow fish at some defined rate, and that rate then defines the required fish feeding rate. The fish feeding rate in turn then defines waste generation loads and oxygen consumption. A convenient way of defining fish growth is based upon a temperature unit approach and some defined number of temperature units to create a unit growth rate, e.g., one inch per month

$$Growth = \frac{T - T_{base}}{TU_{base}} \tag{4.10}$$

Equation 4.10 predicts growth based with units of inches per month. The T_{base} and TU_{base} terms are defined in Table 4.2 and based upon historical observation and analyzing hatchery records. The terms for trout are from Piper et al. (1982) and the tilapia and perch terms are from the author's unpublished data:

Table 4.2 Temperature Growth Units for Trout, Tilapia and Perch (°F)

	Trout	Tilapia	Perch
T_{base}	32	65	50
TU_{base}	28	15	25
T_{max}	72	85	75

Use Eq. 4.10 subject to the limitation that if T is greater than T_{max}, then calculate the growth at T_{max}. Note that excessive temperatures will compromise growth and/or feed conversion.

For example, if we use tilapia and a water temperature of 80°F (26.7°C):

$$Growth\frac{inches}{month} = \frac{80-65}{15} = 1.00\frac{inch}{month}$$

CONDITION FACTOR AND FISH WEIGHT

The weight of fish can be mathematically related to their length by using a term called the condition factor (CF); the bigger the CF, the more weight per unit length. For a given length, the bigger the CF, then more the weight of a particular fish. Each fish species will have an associated CF value to describe expected or normal body condition (see Table 4.3).

Table 4.3 Condition factors for various fish for use in Eq. 4.11 (from Piper* et al. 1982 or author)

Species	CF
Tilapia	750–850
Tilapia < 1 gm	500
Rainbow and Brown Trout	400
Lake Trout	250
Perch	490
Muskellunge	150
Northern Pike	200
Largemouth Bass	450
Walleye	300

*Note: Piper, pg. 406, uses CF's that are 10 times the above values, e.g., Piper would report the trout CF as 4,000, #400 and would divide by 10,000,000 instead of 1,000,000 in Eq. 4.11.

Thus, knowing the CF value is a good tool to gauge current feeding protocols for a given tank of fish. A CF too high means you are over-feeding the fish and a CF too low means you are under-feeding the fish. Using the CF and the fish length, weight can be calculated:

$$\text{WT (lb)} = \frac{CF(L\ inches)^3}{10^6} \tag{4.11}$$

Example - Tilapia that are 7 inches and using a CF of 760,

$$WT\,(7'',\ lb) = \frac{760(7)^3}{10^6} = 0.26\ lb$$

WEIGHT GAIN

Now, we can calculate actual gain by calculating the weights associated with two specific fish lengths over the time period required to achieve this growth. In the previous example, we grew 7 inch tilapia for one month at 80°F, which produced one inch of growth to a size of 8 inches. The new weight is then:

$$WT_{new}\,(7+1)'' = \frac{760(8)^3}{10^6} = 0.39\ lbs$$

The relative efficiency of converting feed energy into animal flesh is described by a term called the feed to gain ratio, or FG. FG is the inverse or feed conversion efficiency. FG is used to calculate feeding rates required to achieve projected growth rates:

$$FG = \frac{feed}{gain} \tag{4.12}$$

$$Feed = FG \cdot gain \tag{4.13}$$

$$\frac{feed}{month} = (\mathrm{WT_{new}} - \mathrm{WT_{old}}) \cdot \mathrm{FG} \tag{4.14}$$

Returning to our example of tilapia increasing in length for 7 to 8 inches in one month, the feed per fish required per month is:

$$\frac{feed}{month} = (0.39\ \mathrm{lb} - 0.26\ \mathrm{lb}) \cdot \mathrm{FG}$$

In the above example, each tilapia will gain 0.13 lb (0.39–0.26 lb = 0.13 lb) over the one month period. Converting this to a daily rate and using 30.5 days per month, the daily gain is:

$$daily\ gain = \frac{0.13\ lb}{month} \cdot \frac{454\ g}{lb} \cdot \frac{month}{30.5\ day} = \frac{1.93\ g}{day} \tag{4.15}$$

The expression in Eq. 4.15 is calculating the average daily gain based upon the average length gain over the month. Since weight gain is cubically related to length gain, the gain for the "last-day" of the month can be calculated based upon the length incremental gain on the last-day of the month. This concept is demonstrated in the following example.

Example

Calculate the maximum daily feeding rate for a tank containing 10,000 tilapia at a harvest weight of 2.00 lb (CF=760; a tank temperature of 80°F; and a FG of 1.1).

Solution

First calculate the length of a 2.00 lb tilapia:

$$L\ (inch) = \left[\ \frac{1{,}000{,}000 \times 2.00}{760}\ \right]^{1/3}$$

$$L\ (inch) = 13.81 \text{ (from Eq. 4.10)}$$

$$Growth \frac{inches}{month} = \frac{80-65}{15} = 1.00 \frac{inch}{month}$$

Daily Increment of length (inch) = 1.00/30.5 = 0.033 inch

$$Wt\ (day-1) = 760\ (13.81 - 0.033)^3/10^6$$

$$Wt\ gain = 2.000\ lb - 1.986$$

Feed /day/fish = 0.014 lb gain (6.4 g)

$$\text{Tank feed/day} = 10{,}000\ \frac{fish}{tank} \cdot 0.014\ \frac{lb\ gain}{fish \cdot day} \cdot \text{FG}\ \frac{lb\ feed}{lb\ gain}$$

$$= 140 \cdot 1.1 = 154 \text{ lb feed per day per tank}$$

Feeding charts can be constructed using the above approach. Obviously, these calculations are well suited for spreadsheet applications.

FG can be very erratic. Assuming good management and good quality feed (protein content of 38 to 42%), guidelines for FG are as follows for tilapia:

0.7 to 0.9 (tilapia < 100 gram)
1.2 to 1.3 (tilapia > 100 gram)

Similarly, given length growth, the weight gain for any particular fish species can be calculated by using the condition factor for the particular species (both the tilapia and perch growth constants were developed by the authors).

Remember, all the "P" terms in Eq. 4.2 are related to the feeding rate term. Essentially, all design aspects of RAS technology are related to or in fact are based upon the daily design feeding rate. The daily design feeding rate is the critical value needed in designing a complete RAS. It should make some intuitive sense then, that one must calculate the largest anticipated feeding rate in order to properly design the water conditioning components of an RAS. And when will this largest feeding rate occur? The answer being when the fish tank reaches its largest biomass and associated fish mass. This is also where many designs fail, since the largest fish mass and largest feeding rate is the most stressful condition on the RAS. Any of the individual water quality parameters can quickly become the defining variable for water quality control, meaning the water flow rate through a particular water conditioning design component is not sufficient to remove the generated pollutant load, e.g., CO_2 removal device. The water parameters always come to an equilibrium based upon a balance between production load, water flow rate and change in concentration for different water quality variables across the conditioning component (see Eq. 4.6). The challenge to the designer and manager is to make sure these equilibrium values at least as "good" as the design target values for these variables.

WET WEIGHT MEASUREMENTS

Both terms in the FG are wet basis, but while feed will typically be from 5 to 10% moisture, the gain term is more nearly 80 to 85% moisture. FG values for broilers, pigs, and beef are something like 2.0, 2.5 and 3.0, respectively, while in fish, the FG value may be less than one. Quite typically FG values will be less than one for fingerlings up to a 100 gram or larger in size. This sometimes confuses people as to how the FG ratio could be less than one, but it is because a comparison of wet weight to dry weight terms is being made.

4.4 DESIGN EXAMPLES

Calculate Growth Period

The following calculation is applicable for calculating the growth period of tilapia from 50 grams to 800 grams, at 82°F; assume a CF = 760.

Calculate length at 50 grams (0.11 lb)

$$L\ (inch\ at\ 50\ g) = \left[\ \frac{1{,}000{,}000 \cdot 0.11}{760}\ \right]^{1/3} = 5.25\ \text{inches}$$

$$L\ (inch\ at\ 800\ g) = \left[\ \frac{1{,}000{,}000 \cdot 1.76}{760}\ \right]^{1/3} = 13.24 \text{ inches}$$

Total change in length required from 50 to 800 grams:

(13.24 –5.25) = 7.99 inches

Calculate growth rate at the specified rearing temperature of 82°F:

$$Growth\frac{inches}{month} = \frac{82-65}{15} = 1.13\frac{inch}{month}$$

Total time to achieve the required length =

$$Growth\ period = \frac{7.99\ inch}{1.13\ \dfrac{inch}{month}} = 7.07 \text{ months}$$

Note that this growth could be forced to occur in a single tank, or the growth could be managed to occur in three tanks that are managed sequentially. If a three tank scheme is used, then the fish would reside in each tank for 1/3 of the total growth time, or

Time in a particular tank = 7.07 months/3 size classes
= 2.36 months/size class
(or 10 weeks in each tank)

EXAMPLE: REQUIRED FLOW RATE DESIGN PROBLEM

Calculate the required design flow rate for a 100% recirculating flow for a design fish feeding rate of 100 kg feed/day @ 38% protein. Calculate the required flow rate for each water quality parameter and then identify the controlling parameter. Assume your location is 228.6 m (750 feet) above sea level. Use the following design target values for water quality:

T^{o} = $28°C$
TSS = 10 mg/L
TAN = 2 mg/L
$Oxygen$ = 5.0 mg/L
CO_2 = 40 mg/L

Assume treatment efficiency for each of the system components as follows:

TSS = 75%
TAN = 35%
Oxygen = 70% but gas concentration is 50% oxygen
CO_2 = 70%

Solution

Given a feeding rate of 100 kg per day of feed with 38% protein, you were to compute the required steady state flow rate for maintaining the following water quality levels: 10 mg/L TSS, 2 mg/L TAN, 5 mg/L O_2, and 40 mg/L CO_2. You were to assume the following efficiencies for the treatment devices: 75% for TSS, 35% for TAN, 70% for O_2, and 70% for CO_2. Additionally, the tank water temperature was set to 28°C, and the gas composition in the O_2 treatment device maintained at 50% O_2.

General Mass Balance

$QC_2 + P = QC_1$ or $Q\,(C_2 - C_1) = -P$ (repeat of Eq. 4.5)
or $Q\,(C_1 - C_2) = P$

Oxygen

$C_2 = C_1 + TE\,(C_{sat} - C_1)$
$C_1 = 5\ mg/L$
$TE = 0.7$

Calculate C_{sat} (Equations from Chapter 8 to determine oxygen saturation)

$T = 28 + 273.15 = 301.15\text{K}$
$\beta = \exp\,[A_1 + A_2\,(100/T) + A_3 \ln\,(T/100)]$
$\beta = \exp\,[-58.3877 + 85.8079\,(100/301.15) + 23.8439\,(\ln\,(301.15/100))]$
$\beta = \exp\,[-58.3877 + 28.4934 + 26.2864]$
$\beta = \exp\,[-3.6079]$
$\beta = 0.02711$ L/L-atm
$BP = 10^{(2.88014 - h/19748.2)}$
$BP = 10^{(2.880814 \quad 228.6/19748.2)}$ *(Height above MSL is 750 ft (228.6m))*
$BP = 10^{(2.880814 \quad 0.011576)}$
$BP = 10^{(2.869238)}$
$BP = 740$ mmHg
$P_{wv} = 28.971$ *(From Eq. 8.6)*

$K = 1.42903$

$X = 0.5$ *(50% oxygen atmosphere in device)*

$C_{sat} = 1{,}000\ K\ \beta\ X\ ((BP - P_{wv})/760)$

$C_{sat} = 1{,}000\ (1.42903)\ (0.02711)\ (0.5)\ ((740-28.971)/760)$

$C_{sat} = 1{,}000\ (1.42903)\ (0.02711)\ (0.5)\ (0.9356)$

$C_{sat} = 18.1$ mg/L

(atmospheric C_{sat} would be 7.57 mg/L; using Eq. 4.4, obtain 8.13 which is high)

$C_2 = 5 + 0.7\ (18.1-5)$

$C_2 = 14.2$ mg/L

Oxygen production (-P) is the sum of fish and bacterial oxygen consumption:

$$\frac{0.25 kg O_2\ Consumed\ by Fish}{kg\ feed} + \frac{0.12 kg O_2\ Consumed\ by Bacteria}{kg\ feed} = \frac{0.37 kg Oxygen}{kg\ feed}$$

$$P = \left(\frac{100\ kg\ feed}{day}\right)\left(\frac{0.37\ kg\ Oxygen}{kg\ feed}\right)\left(\frac{10^6\ mg}{kg}\right) = \frac{37{,}000{,}000\ mg\ Oxygen}{day}$$

Q (14.2 mg/L) + (-37,000,000) mg/day = Q (5 mg/L) *(apply Eq. 4.5)*

$$Q = \frac{\dfrac{37{,}000{,}000\ mg\ Oxygen}{day}}{(14.2-5)\dfrac{mg\ Oxygen}{liter}} = \frac{4{,}021{,}739\ liter}{day}$$

$$Q = \left(\frac{4{,}021{,}739\ liter}{day}\right)\left(\frac{1\ gallon}{3.785\ liter}\right)\left(\frac{1\ day}{1{,}440\ minutes}\right)$$

$$= \underline{\underline{738\ gpm\ required\ for\ O_2\ control}}$$

Carbon Dioxide (CO_2)

$C_2 = C_1 + TE\ (C_{best} - C_1)$

$C_1 = 40$ mg/L
TE = 0.7
$C_{best} = 0.5$ mg/L

$C_2 = 40 + 0.7\ (0.5–40)$
$C_2 = 12.4$ mg/L

Carbon dioxide production (+P) is based on oxygen consumption:

$$\left(\frac{0.37\ kg\ O_2\ Oxygen\ Consumed}{kg\ feed}\right)\left(\frac{1.375\ kg\ CO_2\ Produced}{kg\ Oxygen\ Consumed}\right)$$
$$= \frac{0.50875\ kg\ CO_2}{kg\ feed}$$

$$P = \left(\frac{100\ kg\ feed}{day}\right)\left(\frac{0.50875\ kg\ CO_2}{kg\ feed}\right)\left(\frac{10^6\ mg}{kg}\right) = \frac{50{,}875{,}000\ mg\ CO_2}{day}$$

Q (12.4 mg/L) – 50,875,000 mg/day = Q (40 mg/L) *(apply Eq. 4.5)*

$$Q = \frac{\dfrac{50{,}875{,}000\ mg\ CO_2}{day}}{(40-12.4)\dfrac{mg\ CO_2}{liter}} = \frac{1{,}843{,}297\ liter}{day}$$

$$Q = \left(\frac{1{,}843{,}297\ liter}{day}\right)\left(\frac{1\ gallon}{3.785\ liter}\right)\left(\frac{1\ day}{1{,}440\ minutes}\right)$$
$$= \underline{\underline{338\ gpm\ required\ for\ CO_2\ control}}$$

Total Suspended Solids (TSS)

$C_2 = C_1 + TE\ (C_{best} – C_1)$
$C_1 = 10$ mg/L
TE = 0.75
$C_{best} = 0$ mg/L

$C_2 = 10 + 0.75\ (0–10)$
$C_2 = 2.5$ mg/L

TSS production (+P) is based on feeding rate (assume feed has 0% moisture content):

$$P = \left(\frac{100\ kg\ \ feed}{day}\right)\left(\frac{0.25\ kg\ TSS}{kg\ \ feed}\right)\left(\frac{10^6\ \ mg}{kg}\right) = \frac{25{,}000{,}000\ mg\ TSS}{day}$$

$$Q\ (2.5\ \text{mg/L}) - 25{,}000{,}000\ \text{mg/day} = Q\ (10\ \text{mg/L})\ \textit{(apply Eq. 4.5)}$$

$$Q = \frac{\dfrac{25{,}000{,}000\ mg\ TSS}{day}}{(10 - 2.5)\dfrac{mg\ TSS}{liter}} = \frac{3{,}333{,}333\ liter}{day}$$

$$Q = \left(\frac{3{,}333{,}333\ liter}{day}\right)\left(\frac{1\ gallon}{3.785\ liter}\right)\left(\frac{1\ day}{1{,}440\ minutes}\right)$$
$$= \underline{\underline{612\ gpm\ required\ \ for\ TSS\ control}}$$

Total Ammonia Nitrogen (TAN)

$C_2 = C_1 + TE\ (C_{best} - C_1)$
$C_1 = 2$ mg/L
$TE = 0.35$
$C_{best} = 0$ mg/L

$C_2 = 2 + 0.35\ (0–2)$
$C_2 = 1.3$ mg/L

TAN production (+P) is based on feeding rate (feed has 38% protein content):

$$P = \left(\frac{100\ kg\ feed}{day}\right)\left(\frac{0.38\ kg\ Protein}{kg\ feed}\right)\left(\frac{0.092\ kg\ TAN}{kg\ Protein}\right)\left(\frac{10^6\ mg}{kg}\right)$$
$$= \frac{3{,}496{,}000\ mg\ TAN}{day}$$

Q (1.3 mg/L) – 3,496,000 mg/day = Q (2 mg/L) *(apply Eq. 4.5)*

$$Q = \frac{\dfrac{3{,}496{,}000\ mg\ TAN}{day}}{(2-1.3)\dfrac{mg\ TAN}{liter}} = \frac{4{,}994{,}286\ liter}{day}$$

$$Q = \left(\frac{4{,}994{,}286\ liter}{day}\right)\left(\frac{1\ gallon}{3.785\ liter}\right)\left(\frac{1\ day}{1{,}440\ minutes}\right)$$
$$= \underline{\underline{916\ gpm\ required\ for\ TAN\ control}}$$

REQUIRED FLOW RATES

Water Quality Parameter	Required Flow rate (gpm)
TSS	612
TAN	916*
Oxygen	738
Carbon Dioxide	338

* controlling flow rate

The only footnote to the above is that you must also calculate the required hydraulic loading for the biofilter as in $m^3/hr/m^2$ (gpm/ft^2). This may become the controlling flow rate over and above the flowrates required to maintain water quality conditions. Finally, just remember that if such were the case, it simply means that the equilibrium concentrations for all the water quality parameters would be just a little *better* and the fish will never complain about that

LIST OF SYMBOLS

C_x	Concentration of a particular water quality parameter X, $\frac{kg\ of\ X}{10^6\ kg\ water}$
C_0, C_1 and C_2	Concentrations of parameter X crossing the control volume, mg/L
CF	Condition factor that relates fish weight and length, dimensionless
D_{fish}	Fish density in culture vessel, kg/m^3
F	Feeding rate, kg/day
$F_{\%}$	Feeding rate per day as a percentage of fish body mass, %
FG	Feed to gain ratio, dimensionless
L	Loading rate, kg/m^3/hr
L_{oxygen}	Allowable fish biomass loading due to oxygen availability, kg/ Lpm
PC	Protein concentration or feed, decimal
P_{Oxygen}	Production rate (negative as related to feed consumption) of oxygen, kg/day
P_{CO2}	Production rate of carbon dioxide, kg/day
P_{TAN}	Production rate of total ammonia nitrogen, kg/day
$P_{Solids,\ TSS}$	Production rate of total suspended solids, kg/day
Q_0	Flow rate passing through culture tank (discharge), m^3/day (as kg/day)
Q_1	Water that is recirculated, kg/day
R	Number of water exchanges per hour through the rearing unit, hr^{-1}
t	Time period for feeding daily ration, day
TAN	Total Ammonia Nitrogen, kg
TSS	Total Suspended Solids, kg
T_{base}	Equivalent to the temperature at which no growth occurs, °F (°C)
T_{eff}	Treatment efficiency (removal or addition), %
TU_{base}	Temperature units required to produce an increment of growth per unit time, °F (°C)